Comparison of Stepwise and Piecewise Linear Models of Congestion Avoidance Algorithm

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Motivation and Problem Statement

- Internet needs a tool to control its performance and resource sharing
- This service is provided at the end-to-end level by Transmission Control Protocol (TCP)
- TCP performs distributed flow control. It controls **performance** of the network to prevents it from **congestion** collapse.

What are **qualitative metrics** of congestion control performance?

- Estimations of average TCP throughput
- Estimations of congestion window size

Distributed Flow Control

- Data delivery control is done by **sliding window** and explicit **acknowledgments**
- Sliding window is amount of data which sender is allowed to inject in the network without acknowledgment
- Flow control means control on sliding window size. TCP uses set of algorithms to control its window size W

Additive Increase Multiplicative Decrease Algorithm (AIMD)

$$
\begin{array}{lcl}
W & \xrightarrow{\text{delivery}} & W + 1 \\
\downarrow \text{loss} & & \\
W/2 & &
\end{array}
$$

Details on Sliding Window Size

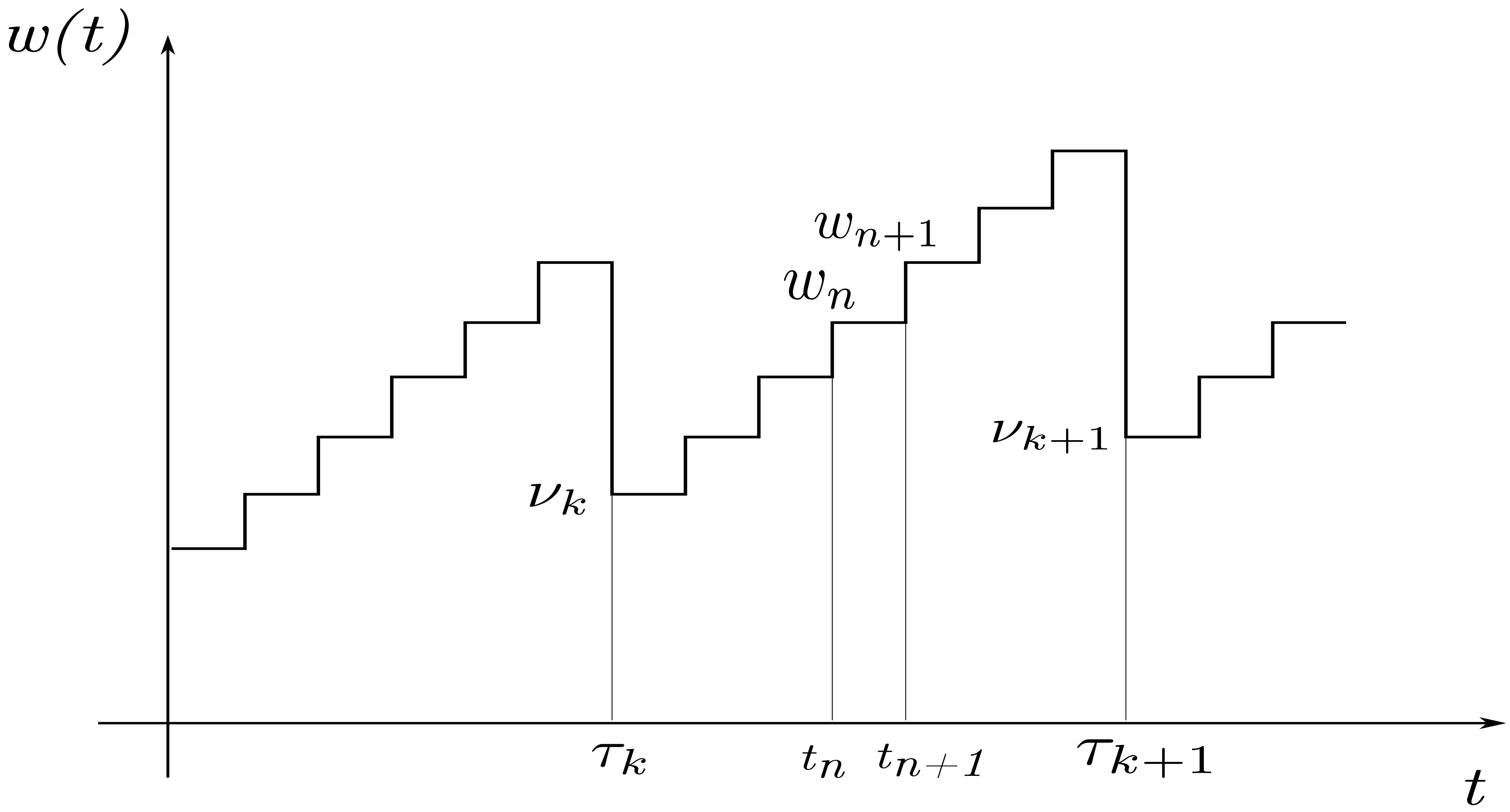

Figure 1: Evolution of TCP congestion window size

Ergodic properties

Let $w(t)$ be cwnd size, t_n are AIMD-rounds end-points, and RTT durations $\xi_n = t_n - t_{n-1}$ are i.i.d. with distribution $R(x)$ and $\mathsf{E}[\xi_n] < \infty$. Data segment losses form Poisson flow $\lambda > 0$ and $\{w_n = w(t_n)\}$. Denote

$$q = \int_0^\infty e^{-\lambda\tau} dR(\tau) \tag{1}$$

the probability that there were no segment losses on the interval $[t_{n-1}, t_n]$.

Theorem 1 *Markov chain $\{w_n\}$ has steady state distribution.*

Theorem 2 *If $\exists\ \epsilon > 0 : \quad R(\epsilon) \leq 1 - \epsilon$ then the random process $\{w(t)\}_{t>0}$ is ergodic.*

Ergodic properties

Let τ_k be equal to the first moment t_n, arrived after kth data loss event,

$$\tau_k = t_n : \; w(t_n + 0) = \left\lfloor \frac{w(t_n)}{2} \right\rfloor, \; k = 1, 2. \ldots$$

and $\tau_{k_1} < \tau_{k_2}$, if $k_1 < k_2$. The sequence $\{\nu_k = w(\tau_k + 0)\}_{k \geq 0}$ is the Markov chain embedded in the Markov chain $\{w_n\}$. Lets denote

$$f_j = \mathbf{P}\left\{\nu_{k+1} = \left\lfloor \frac{1}{2}\left(\left\lfloor \frac{\nu_k}{2} \right\rfloor + j\right) \right\rfloor\right\}, \; j = 0, 1 \ldots . \qquad (2)$$

Let's define the expectation determined by f_j as $B = \sum_{j=1}^{\infty} j f_j$.

Theorem 3 *If B is finite then the Markov chain $\{\nu_k\}$ has steady state distribution.*

Kolmogorov Equations

For $\{w(t)\}_{t>0}$ loss events form the sequence of Bernoulli tries with parameter q,

$$f_j = (1-q)q^j$$

$$\begin{aligned} p_0 &= (1-q)p_0 + (1-q)p_1 \\ p_m &= qp_{m-1} + (1-q)p_{2m} + (1-q)p_{2m+1} \qquad m > 1 \end{aligned} \tag{3}$$

for the Markov chain $\{w_n\}$ and

$$\begin{aligned} \pi_0 &= \pi_0(f_0 + f_1) + \pi_1 f_0 \\ \pi_r &= \sum_{j=0}^{2r} \pi_j f_{2r-j} + \sum_{j=0}^{2r+1} \pi_j f_{2r+1-j} \end{aligned} \tag{4}$$

for the Markov chain $\{\nu_k\}$.

Kolmogorov Equations

Theorem 4 *Let the sequence* $\{\pi_r\}$ *is the solution of the equations (4), then*

$$p_m = \sum_{j=0}^{m} \pi_j q^{m-j}. \tag{5}$$

Let $W = \lim_{t\to\infty} \mathsf{E}[w(t)]$.

Corollary 1

$$W = \lim_{k\to\infty} \mathsf{E}[\nu_k] + B. \tag{6}$$

Main Result

$$\pi_r = \sum_{j=0}^{2r} \pi_j (f_{2r-j-1} + f_{2r-j}), \;\; f_{-1} = 0 \tag{7}$$

$$\sum_{i=0}^{\infty} \pi_i z^{2i} = \left(\sum_{j=0}^{\infty} \pi_{2j} z^{2j} \right) \left(\sum_{k=0}^{\infty} (f_{2k} + f_{2k+1}) z^{2k} \right) + \\ + \left(\sum_{j=0}^{\infty} \pi_{2j+1} z^{2j+1} \right) \left(\sum_{k=0}^{\infty} (f_{2k-1} + f_{2k}) z^{2k-1} \right) \tag{8}$$

or

$$P(z^2) = \frac{1+z}{2z} F(z) P(z) + \frac{z-1}{2z} F(-z) P(-z) \tag{9}$$

Main Result

Since $|P(-1)| \leq 1$, then using the corollary of the theorem 4 one obtains

$$2F'(1) - \frac{1}{2}(|F(-1)| + 1) \leq W \leq 2F'(1) + \frac{1}{2}(|F(-1)| - 1) \qquad (10)$$

The Example

Lets $\xi_n = d$ deterministic variable and $\lambda d < 1$. Then $q = e^{-\lambda d}$ and

$$F(z) = \frac{1 - e^{-\lambda d}}{1 - ze^{-\lambda d}}$$

Compare with

Altman E., Avrachenkov K., Barakat C. A Stochastic model of TCP/IP with Stationary Random Losses, Proceedings of ACM SIGCOMM'00. Stockholm, 2000. pp. 231-242.

$$\mathsf{E}[X_n] = \frac{\alpha}{\lambda(1-\nu)} \tag{11}$$

α is a rate of the window growth in the absence of random losses, ν is multiplicative decrease factor and λ is the loss intensity.

The example

Following stepwise model described above one sets $\alpha = 1/d$ and $\nu = 1/2$.

$$2F'(1) \approx \frac{2}{\lambda d} \tag{12}$$

and $-0,5 < \frac{1}{2}\left(|F(-1)| - 1\right) < 0$.

Thus if RTT variability is low and segment losses form Poisson flow, then piecewise linear model is **first order approximation** of the stepwise model.

Conclusion

- The stepwise model of AIMD New Reno congestion avoidance is analyzed.
- The semi-Markovian random process is formulated, theorems on its ergodic properties are proved.
- The functional equation which defines generating function of Markov chain embedded in the semi-Markovian process is obtained.
- The estimations of upper and lower bounds of the steady state expectation of the congestion window size is formulated.
- The interval estimation obtained treats RTT as i.i.d. random variables.